DISTORTION-ENERGY THEORY FOR DUCTILE MATERIALS

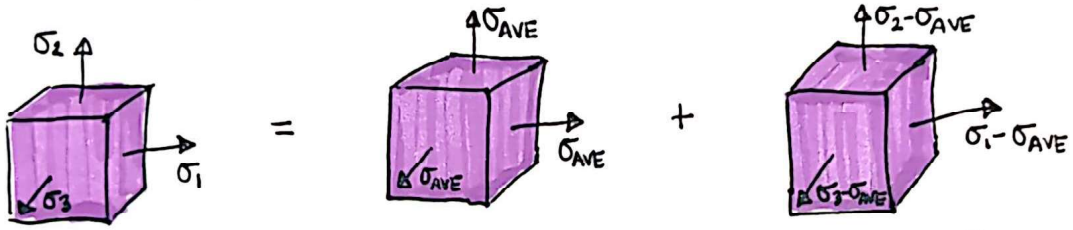
THE **DISTORTION-ENERGY (DE) THEORY** PREDICTS THAT YIELDING OCCURS WHEN THE DISTORTION STRAIN ENERGY PER UNIT VOLUME REACHES OR EXCEEDS THE DISTORTION STRAIN ENERGY PER UNIT VOLUME FOR YIELD IN SIMPLE TENSION OR COMPRESSION OF THE SAME MATERIAL.
(ALSO KNOWN AS THE **VON MISES** OR **VON MISES-HENCKY THEORY**)

CONSIDER A STRESS ELEMENT SUBJECTED TO PRINCIPAL STRESSES $\sigma_1, \sigma_2, \sigma_3$.
THESE PRINCIPAL STRESSES CAN BE SEPARATED INTO 2 COMPONENTS:

↳ THE HYDROSTATIC STRESS CAUSES ONLY VOLUME CHANGE.

$$\sigma_{AVE} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

↳ THE DISTORTIONAL COMPONENT CAUSES ONLY ANGULAR DISTORTION.



TRIAXIAL STRESSES | HYDROSTATIC COMPONENT | DISTORTIONAL COMPONENT

THE DE THEORY SUGGESTS THAT IT IS THE DISTORTIONAL COMPONENT THAT LEADS TO FAILURE. THE FAILURE CRITERION IS

$$n = \frac{S_y}{\sigma'}$$ WHERE σ' IS THE **VON MISES STRESS**

THE VON MISES STRESS IS

$$\sigma' = \sqrt{\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}{2}}$$

$$\sigma' = \sqrt{\frac{(\sigma_x-\sigma_y)^2 + (\sigma_y-\sigma_z)^2 + (\sigma_z-\sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2)}{2}}$$

FOR A PLANE STRESS STATE,

$$\sigma' = \sqrt{\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2}$$

MAXIMUM-SHEAR-STRESS THEORY FOR DUCTILE MATERIALS

THE **MAXIMUM-SHEAR-STRESS (MSS) THEORY** PREDICTS THAT YIELDING BEGINS WHEN THE MAXIMUM SHEAR STRESS IN ANY ELEMENT EQUALS OR EXCEEDS THE MAXIMUM SHEAR STRESS IN A TENSION-TEST SPECIMEN OF THE SAME MATERIAL WHEN THAT SPECIMEN BEGINS TO YIELD. (ALSO KNOWN AS THE TRESCA OR GUEST THEORY)

THE FAILURE CRITERION FOR THE MSS THEORY IS:

$$n = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{S_y}{2\tau_{max}}$$

where n = SAFETY FACTOR. REMEMBER, WHEN $n=1$ FAILURE IS PREDICTED.

and $\sigma_1 \geq \sigma_2 \geq \sigma_3$ ARE THE ORDERED PRINCIPAL STRESSES.

FOR A PLANE STRESS STATE, ONE OF THE PRINCIPAL STRESSES IS ZERO. THERE ARE 3 POSSIBILITIES:

(1) $\sigma_A \geq \sigma_B \geq 0 \quad (\sigma_1 \geq \sigma_2 \geq 0)$

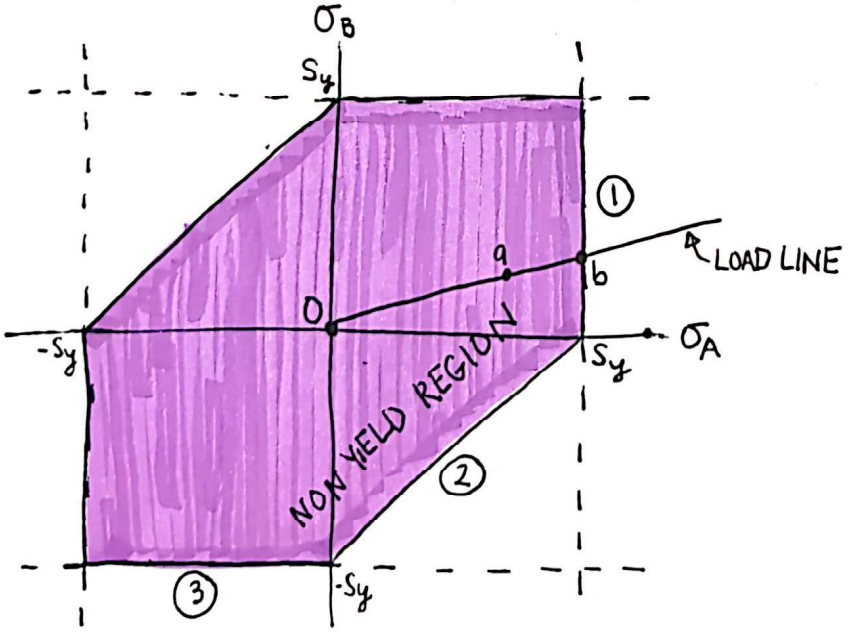
THEN, THE MSS FAILURE CRITERION IS $\sigma_A \geq S_y \quad \left(n = \frac{S_y}{\sigma_A} = \frac{S_y}{\sigma_1}\right)$

(2) $\sigma_A \geq 0 \geq \sigma_B \quad (\sigma_1 \geq 0 \geq \sigma_3)$

THEN, THE MSS FAILURE CRITERION IS $\sigma_A - \sigma_B \geq S_y \quad \left(n = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{S_y}{\sigma_A - \sigma_B}\right)$

(3) $0 \geq \sigma_A \geq \sigma_B \quad (0 \geq \sigma_2 \geq \sigma_3)$

THEN, THE MSS FAILURE CRITERION IS $\sigma_B \leq -S_y \quad \left(n = \frac{S_y}{-\sigma_3} = \frac{S_y}{-\sigma_B}\right)$



* IF POINT a REPRESENTS THE STRESS STATE OF A CRITICAL STRESS ELEMENT OF A MEMBER, AND POINT b REPRESENTS THE STRESS STATE OF THAT SAME ELEMENT AT THE CRITICAL LOAD, THEN THE FACTOR OF SAFETY GUARDING AGAINST YIELD AT POINT a IS

$$n = \frac{Ob}{Oa}$$

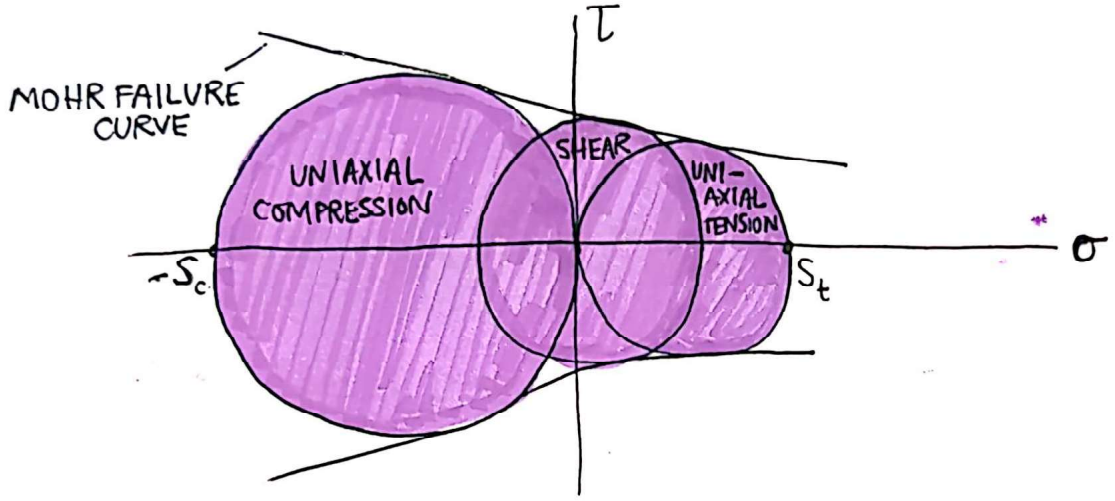
COULOMB-MOHR THEORY FOR DUCTILE MATERIALS

THE DUCTILE COULOMB-MOHR (DCM) THEORY APPLIES TO DUCTILE MATERIALS WITH DIFFERENT COMPRESSIVE & TENSILE STRENGTHS.

- ↳ FOR SOME MAGNESIUM ALLOYS, S_{ut} IS ABOUT TWICE S_{uc}.
- ↳ FOR GRAY CAST IRONS, S_{uc} IS 3-4 TIMES S_{ut}

THE MOHR THEORY WAS DEVELOPED AS A GRAPHICAL METHOD (~1900s)

- ↳ BASED ON 3 SIMPLE TESTS: TENSION, COMPRESSION, & SHEAR
- ↳ THE FAILURE CURVE IS TANGENT TO THE 3 MOHR'S CIRCLES.



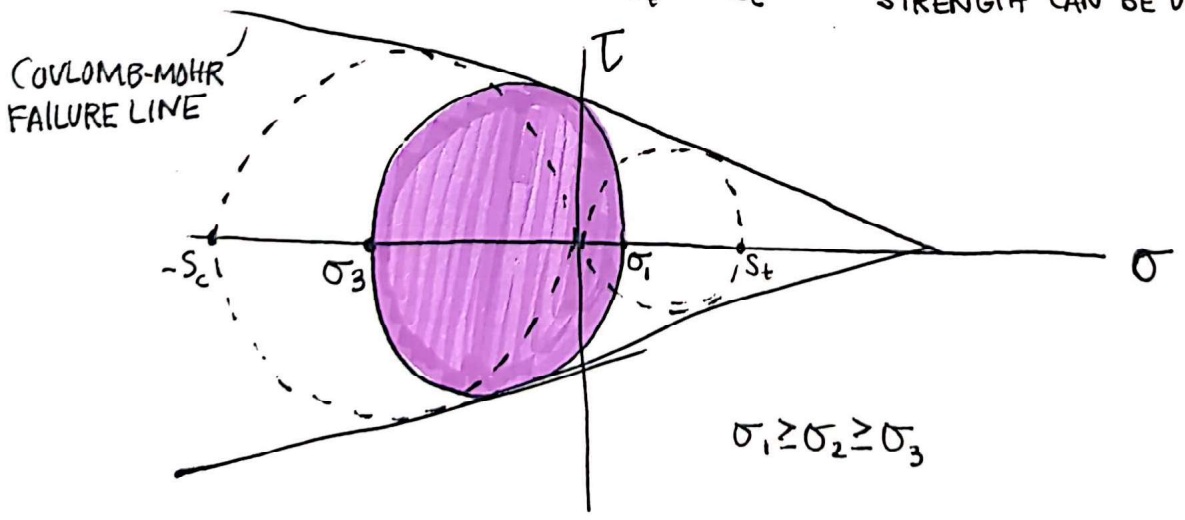
A VARIATION OF MOHR'S THEORY, CALLED THE COULOMB-MOHR THEORY IS BASED ON 2 SIMPLE TESTS: TENSION & COMPRESSION

- ↳ THE FAILURE CURVE IS ASSUMED TO BE A STRAIGHT LINE.

THE FAILURE CRITERION FOR THE DCM THEORY IS:

$$\frac{1}{n} = \frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c}$$

(WHERE EITHER YIELD OR ULTIMATE STRENGTH CAN BE USED)



FOR A PLANE STRESS STATE, ONE OF THE PRINCIPAL STRESSES IS ZERO. THERE ARE 3 POSSIBILITIES:

① $\sigma_A \geq \sigma_B \geq 0$ $(\sigma_1 \geq \sigma_2 \geq 0)$

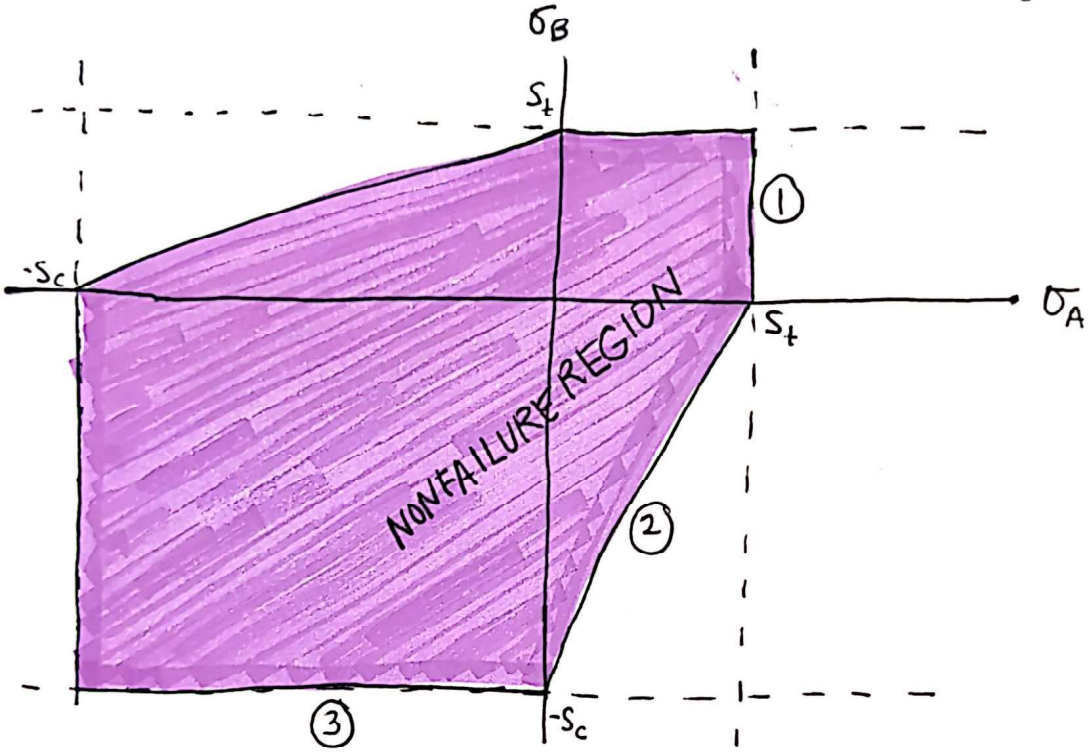
THEN, THE FAILURE CRITERION IS $\sigma_A \geq S_t$ $\left(n = \frac{S_t}{\sigma_A} = \frac{S_t}{\sigma_1}\right)$

② $\sigma_A \geq 0 \geq \sigma_B$ $(\sigma_1 \geq 0 \geq \sigma_3)$

THEN, THE FAILURE CRITERION IS $\frac{\sigma_A}{S_t} - \frac{\sigma_B}{S_c} \geq 1$ $\left(\frac{1}{n} = \frac{\sigma_A}{S_t} - \frac{\sigma_B}{S_c} = \frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c}\right)$

③ $0 \geq \sigma_A \geq \sigma_B$ $(0 \geq \sigma_2 \geq \sigma_3)$

THEN, THE FAILURE CRITERION IS $\sigma_B \leq -S_c$ $\left(n = \frac{S_c}{-\sigma_B} = \frac{S_c}{-\sigma_3}\right)$



COMPARE THIS TO THE MSS & DE THEORY YIELD ENVELOPES FOR PLANE STRESS: